Grade 2 Mathematics, Quarter 2, Unit 2.1

Develop an Understanding of Linear Measurement by Using Standard Units and Estimating Length

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Measure length using a variety of tools: ruler, yardstick, meter stick, and measuring tapes.
- Measure length twice using length units of different length.
- Estimate lengths in inches, feet, centimeters, and meters.
- Determine the difference between two lengths and express the difference in terms of a standard unit length.

Mathematical practices to be integrated

Use appropriate tools strategically.

- Select from a variety of measuring tools based upon the length to be measured and units of measurement to be used.

Reason abstractly and quantitatively.

- Consider units involved.
- Attend to the meaning of quantities.

Essential questions

- How can you determine the appropriate tool to measure a given object?
- What unit of measure should you use to find the length of _____(your foot, the classroom)?
- How can you use a table to help you find how many inches are in ____ feet?
- What strategy can you use to find how many meters there are in _____ centimeters?

Written Curriculum

Common Core State Standards for Mathematical Content

Measurement and Data **2.MD**

Measure and estimate lengths in standard units.

2.MD.1 Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

2.MD.2 Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.

2.MD.3 Estimate lengths using units of inches, feet, centimeters, and meters.

2.MD.4 Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Clarifying the Standards

Prior Learning

In kindergarten, students directly compared two objects to determine which was longer or shorter. In first grade, students used a third object to compare the length of two objects. They also used multiple copies of an object as a length unit, laying it end to end to determine the number of units that would cover the object with no gaps or overlap.

Current Learning

At the developmental level, second-grade students use a variety of tools to measure objects and determine the most appropriate tool for a given object. They compare different units of measure (i.e., inches, feet, yards, meters, centimeters). Students estimate lengths of objects in a variety of units and determine the difference in length between objects.

Routines include adding 1, adding 10, and using strategies to add facts to 20.

Future Learning

In third grade, students will collect data in lengths. They will use rulers marked to halves and fourths of an inch. Third-graders will also measure intervals of time, volume, and use those measurements to solve one-step word problems.

Additional Findings

According to *Principles and Standards for School Mathematics*, in grade 2, all students should recognize the attributes of length, volume, weight, area, and time. They should understand measurable attributes of objects as well as the units, systems, and processes of measurement. They should apply appropriate techniques, tools, and formulas to determine measurements (pp. 102–106).

A Research Companion to Principles and Standards for School Mathematics states, "when measuring area, many students do not understand that area measures require identical units. Students often cannot structure a rectangle as an array of units to measure area. Students in early elementary grades have a wide range of misconceptions of array" (p. 185).

Grade 2 Mathematics, Quarter 2, Unit 2.2

Develop an Understanding of Place Value to 1,000

Overview

Number of instructional days: 12 (1 day = 45–60 minutes)

Content to be learned

- Demonstrate conceptual understanding of whole numbers from 0–900 using place value by applying concepts of equivalency in composing and decomposing.
- Apply expanded notation to demonstrate equivalency using models, explanations, or other representations.
- Demonstrate understanding of relative magnitude of numbers from 0–900 by ordering whole numbers.
- Compare whole numbers to each other or to benchmark numbers.
- Demonstrate understanding of the relation of inequality when comparing whole numbers using 1, 10, or 100 more and 1, 10, or 100 less.
- Connect number words and numerals to the quantities (0-1000) they represent using models, number lines, or explanations.

Mathematical practices to be integrated

Model with mathematics.

- Make and draw models of three-digit numbers.

Look for and make use of structure.

- Decompose numbers into hundreds, tens, and ones.
- Use patterns to skip count by 5s, 10s, and 100s.
- Use patterns to add 10 and 100 or subtract 10 and 100.

Essential questions

- How can you make a number that is 10 more? 10 less? 100 more? 100 less?
- How many different ways can you write (a three-digit number) in expanded notation?
- How could you compare any set of numbers?
- How would you order a given set of numbers?
- Why do we use plus signs (+) when we write a number in expanded form?
- How can you decompose/compose (a given number)?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations in Base Ten 2.NBT

Understand place value.

2.NBT.1 Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:

b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

2.NBT.2 Count within 1000; skip-count by 5s, 10s, and 100s.

2.NBT.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

2.NBT.4 Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

2.NBT.8 Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

In kindergarten, students were exposed to the concept of bundling 10 as they counted up to 19 objects. They compared quantities of objects and numerals up to 10. In first grade, students counted bundles of 10, and counted and wrote numerals up to 120. They were introduced to the symbols > and < and used them to compare two-digit numbers.

Current Learning

In this unit, second-grade students are working at the developmental level to expand upon their earlier work in place value by working with quantities up to 1,000.

Future Learning

In the third grade, instruction will shift from addition and subtraction to multiplication and division. Addition and subtraction will be used to solve multi-step problems.

Additional Findings

According to *Principles and Standards for Schools Mathematics*, "understanding of numbers develops in pre-kindergarten through grade 2 as children count and learn to recognize 'how many' in sets of objects. A key idea is that a number can be decomposed and thought about in many ways." On page 32, the book states "instructional programs … should enable all students to understand numbers, ways of representing numbers, relationships among numbers, and number systems; understanding meanings of operations and how they relate to one another; compute fluently and make reasonable estimates" (pp. 32–33).

The book also states that concrete models can help students represent numbers and develop number sense; they can also help bring students' use of written symbols and can be useful in building place value concepts. Using rote materials does not ensure understanding. Teachers need to uncover students' thinking as they work with concrete materials by using appropriate questions. Teachers need to choose interesting tasks that engage students into a deeper level of thinking and reasoning. "It is absolutely essential that students develop a solid understanding of the base-ten numerations system and place-value concepts by the end of second grade" (p.81).

Grade 2 Mathematics, Quarter 2, Unit 2.3

Count Money to Solve Money Problems

Overview

Number of instructional days: 6 (1 day = 45–60 minutes)

Content to be learned

- Solve word problems involving quarters, dimes, nickels, and pennies.
- Use monetary symbols ($, ¢) appropriately.

Mathematical practices to be integrated

Look for and make use of structure.

- Determine the pattern or structure of coins/money.

Look for and express regularity in repeated reasoning.

- Use repeated calculations when adding the same coins together.

Essential questions

- How many ways can you write a given monetary value (dollar notation)?
- How can you find all the different ways to make_____(given monetary value)?
- If you spend ____, how much change will you get back from a dollar?
- How do you know when your answer is reasonable?
- Which strategy do you use to count a given set of coins?
- How can you find the value of a given set of coins?

Written Curriculum

Common Core State Standards for Mathematical Content

Measurement and Data **2.MD**

Work with time and money.

2.MD.8 Solve word problems involving ~~dollar bills~~, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*

Common Core Standards for Mathematical Practice

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

Students in kindergarten and grade 1 are not expected to engage in formal instruction in counting and using coins and bills. Children will enter second grade with their own experiential knowledge.

Current Learning

Second grade is the first time students receive formal instruction in counting coins and bills. Students build upon skip-counting skills practiced in earlier units to count quarters, nickels, dimes, and pennies. Dollar bills are introduced in the fourth quarter. Counting coin combinations is developmental and will need to be reinforced in future routines.

Routines: Skip counting by 5s and 10s.

Future Learning

There are no specific standards addressing money in the third grade. Money may come up in the context of solving problems. In fourth grade, students work in money will be in the context of decimals.

Additional Findings

Benchmark for Science Literacy states, "by the end of second grade, students should know that money can buy things that people need or want. People earn money by working at a job making or growing things, selling things, or doing things to help other people" (p. 168).

The book also states, "From the start, children study numbers and shapes and simple operations on them and do so in as many different contexts as possible. By the end of second grade, students should know… numbers can be used to count any collection of things" (p. 26).

According to *Principles and Standards for School Mathematics*, "K–2 teachers should help students recognize that solving one kind of problem is related to solving another kind of problem" (p. 83).

The book also states, "In pre–K through the 2nd grade, students should understand numbers, ways of representing numbers, relationships among numbers, and number systems; [they should] count with understanding and recognize, 'how many in sets of objects'" (p. 83).

A Research Companion to Principles and Standards for School Mathematics states, "Experience with addition, subtraction, multiplication, and division situations and with the language involved in them allows students to build a mathematically adequate understanding of the operations" (p. 69).

Grade 2 Mathematics, Quarter 2, Unit 2.4

Apply Addition and Subtraction Strategies within 100 to Solve Problems

Overview

Number of instructional days: 12 (1 day = 45–60 minutes)

Content to be learned

- Simplify computations involving whole numbers to solve problems.
- Demonstrate conceptual understanding of equality by finding the value that makes an open sentence true.
- Explore problems of different types: join, separate, part-part-whole, and compare.
- Add up to four two-digit numbers using place value strategies.
- Use concrete models, drawings, and strategies based on place value to solve problems.
- Demonstrate the properties of numbers in addition and subtraction including odd/even numbers and fact families.
- Use knowledge of properties of addition to determine the order that the addends will be added.
- Compose and decompose numbers to solve problems.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Attend to the meaning of quantities.
- Represent a problem situation symbolically.
- Make meaning of numbers and symbols.

Construct viable arguments and critique the reasoning of others.

- Construct arguments using objects, drawings, diagrams and actions.

Essential questions

- What strategies do you use to add more than two 1-digit numbers?
- How do you communicate your thinking (about a given problem) using words, numbers, and/or pictures?
- What counting strategies can you use to solve problems with combinations?
- What strategies do you use to solve a two-digit addition or subtraction problem?
- How can addition help you solve a subtraction problem?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations in Base Ten 2.NBT

Use place value understanding and properties of operations to add and subtract.

2.NBT.6 Add up to four two-digit numbers using strategies based on place value and properties of operations.

2.NBT.7 Add and subtract ~~within 1000~~, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

Operations and Algebraic Thinking 2.OA

Represent and solve problems involving addition and subtraction.

2.OA.1 Use addition and subtraction within 100-to solve one- and two-step-word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations ~~with a symbol for the unknown number to represent the problem.~~[1]

[1] See Glossary, Table 1.

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Clarifying the Standards

Prior Learning

In first grade, students used a variety of models to add within 100. They demonstrated fluency when adding and subtracting facts to 10. They were exposed to a variety of word problems.

Current Learning

During this unit, second grade students are newly introduced to two-step word problems. They continue to work with problems of all types: adding to, taking from, putting together, taking apart and comparing. Unknowns are located in all positions. Students are expected to write equations to reflect their thinking. In the third quarter, students will be expected to write equations with an unknown to represent the problem.

Routines include counting coins, adding 1, adding 10, and using strategies to add facts to 20.

Future Learning

Third-grade instruction will shift from addition and subtraction to multiplication and division, and addition and subtraction will be used to solve multi-step problems. Students will continue to use number lines to solve problems moving from length in second grade to more abstract (e.g., time intervals in minutes).

Additional Findings

According to *A Research Companion to Principles and Standards for School Mathematics*, "research has indicated that beginning with problem situations yields greater problem-solving competence and equal or better computational competence. Children who start with problem situations directly model solutions to these problems. They later move to more advanced mathematical approaches as they progress through levels of solutions and problem difficulty. Thus, their development of computational fluency and their acquisition of problem-solving skills are intertwined as both develop with understanding" (p. 68).

The book also discusses addition and subtraction strategies taught around the world and how children progress through these methods to fact mastery. It states, "analyses of these methods reveal that learners build later methods from earlier methods by chunking, recognizing, and eliminating redundancies; using parts instead of entire methods; and using their knowledge of specific numbers … when these more advanced methods are not supported in the classroom, however, several years separate the earliest and latest users of advanced methods. In contrast, helping children progress through methods can lead all first-graders to methods that are efficient enough to use for all later multi-digit calculations" (pp. 72–76).